The JCSDA Community Radiative Transfer Model (CRTM)

CRTM team:
Benjamin T. Johnson (Team Lead, UCAR/JCSDA)
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With essential contributions from: Barbara Scherllin-Pirscher, Quanhua Liu, Emily Liu, Andrew Collard, Fuqing Zhang, Ping Yang, Kwo-Sen Kuo, *and many others.*

What is the CRTM?

CRTM is the "Community Radiative Transfer Model"

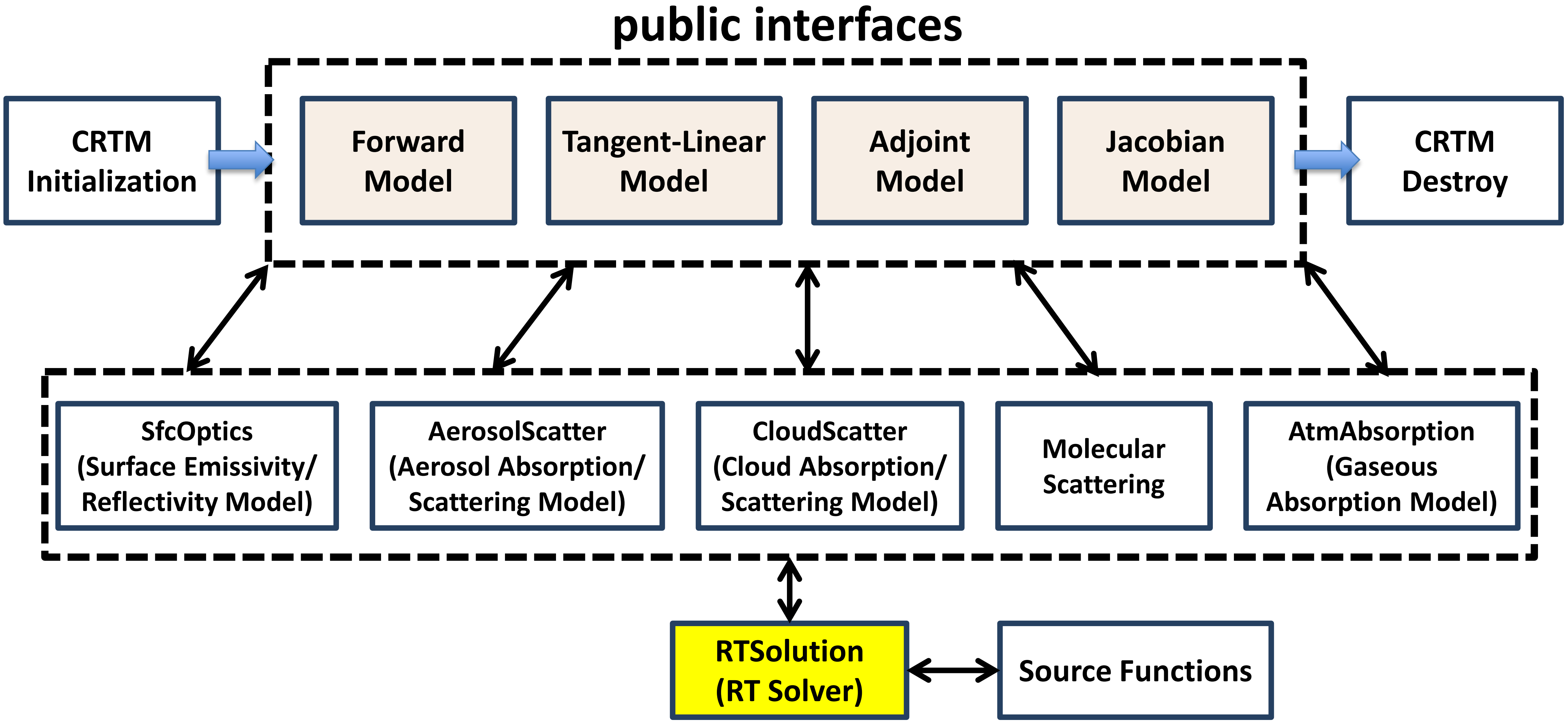
Goal: Fast and accurate community radiative transfer model to enable assimilation of satellite radiances under all weather conditions

Type: 1-D, plane-parallel, multi-stream matrix operator method, advanced method of moments solver, with specular and non-specular surface reflections.

Has aerosol (GO-CART), cloud (2 species), precipitation (4 species); with unpolarized scattering and absorption. Computes gaseous absorption/emission for 6 gaseous species (ODPS).

History: Originally developed (as CRTM) around 2004 by Paul van Delst, Yong Han, Fuzhong Weng, Quanhua Liu, Thomas J. Kleespies, Larry M. McMillin, and many others. CRTM Combines many previously developed models into a community framework, and supports forward, tangent linear, adjoint, and k-matrix modeling of emitted/reflected radiances, with code legacy going back to the mid 1970s (e.g., OPTRAN: McMillin).

CRTM Structure



CRTM Overview

CRTM 1: The first task is an umbrella for all **management, external coordination/collaboration, release support, and oversight of the CRTM team activities** -- covering all versions of CRTM. This specifically includes user-support, documentation, education, and outreach elements.

CRTM 2: The second task is primarily a **software engineering**-driven task aimed specifically at improving the computational aspects of CRTM.

CRTM 3: The third and final task aims at **scientific development and testing**. CRTM users require fast computations of radiances with the highest degree of accuracy and sensitivity possible, while still maintaining the operational computational resource requirements.

Active Areas of Research and Development

Version 2.3.1 progress (Task 1)
Version 3.0.0 progress (Tasks 1,2,3)
CRTM Scattering Indicator, code optimization and solver testing (Task 2)
Community Hydrometeor Model (CHYM) progress (Task 3)
Community Active Sensor Module (CASM) progress (Task 3)
Community Surface Emissivity Model (CSEM) progress (Task 3)
JCSDA Inter-group coordination (JEDI, NIO, SOCA)

CRTM 1: CRTM REL-3.0.0 Plans

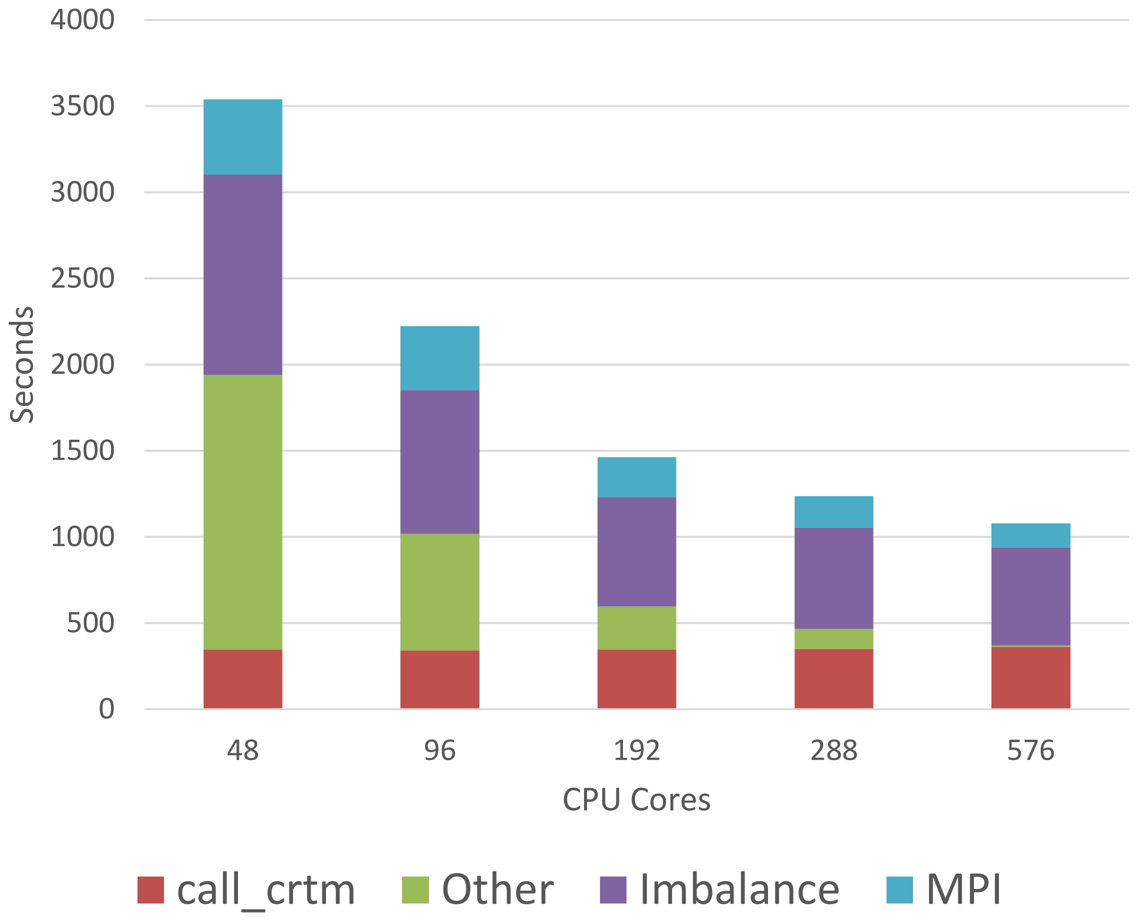
- **Optimization** [J. Rosinski]
  - Thread via OpenMP the loop over observations in GSI routine setuprad.f90
  - Redesign CRTM data structures to enable vectorization of inner loops (vertical dependencies within inner loops prevent this now)
  - CRTM-OSS implementation [AER/T. Zhu]
- **New Solver(s)** [T. Greenwald, Q. Liu, P. Stegmann]
  - **Full Polarization support (optimization / vectorization)**
  - Continue testing of the solvers, include full pol s
  - Add scattering indicator routine to the CRTM to help characterize the degree of scattering in a given atmospheric profile
  - Conduct benchmark timing/accuracy tests using NWP model data and determine optimum configuration for solvers
  - **UV simulation support** [via C. Cao / STAR]

CRTM 1: CRTM REL-3.0.0 Plans

- **Physical properties / ongoing work toward version 3.0**
  - **Aerosols**: Develop new scattering tables to conform with updated aerosol properties (PSD, index of refraction, etc.) in coordination with the "A-Team". Determine whether CMAQ adequately covers NAAPS species; NRL + others will provide an intercomparison of AOD/aerosol speciation operators from GoCart, NAAPS, CMAQ and NGAC.
  - **CSEM**: Continue the CRTM-CSEM integration efforts so that we may have a working version of the integrated CRTM-CSEM package as soon as possible for demonstration and various testing purposes. The implementation of new CSEM functionality and components will depend on the priority and the requirements of the user community.
  - **Microphysics / CHYM**: Continue expanding microphysics database, and testing newly-created scattering tables in stand-alone CRTM and in GSI/GFS for analysis and forecast impact assessment.
  - **Shortwave / IR improvements:** New Hire via Hurricane Supplemental Funding
  - **Active Sensor support:** Lidar and Radar operators are under development, and are expected to be available in V3.0
- **CRTM 3.0 alpha release Q4FY18.**
  - Targeting core functionality (polarization and optimization)

CRTM Optimization (J. Rosinski, JCSDA)

T670 DA time: 48 MPI, various thread counts (1,2,4,6,12) node counts=(2,4,8,12,24) Note: "Other" is a residual calculation from max values, thus an underestimate



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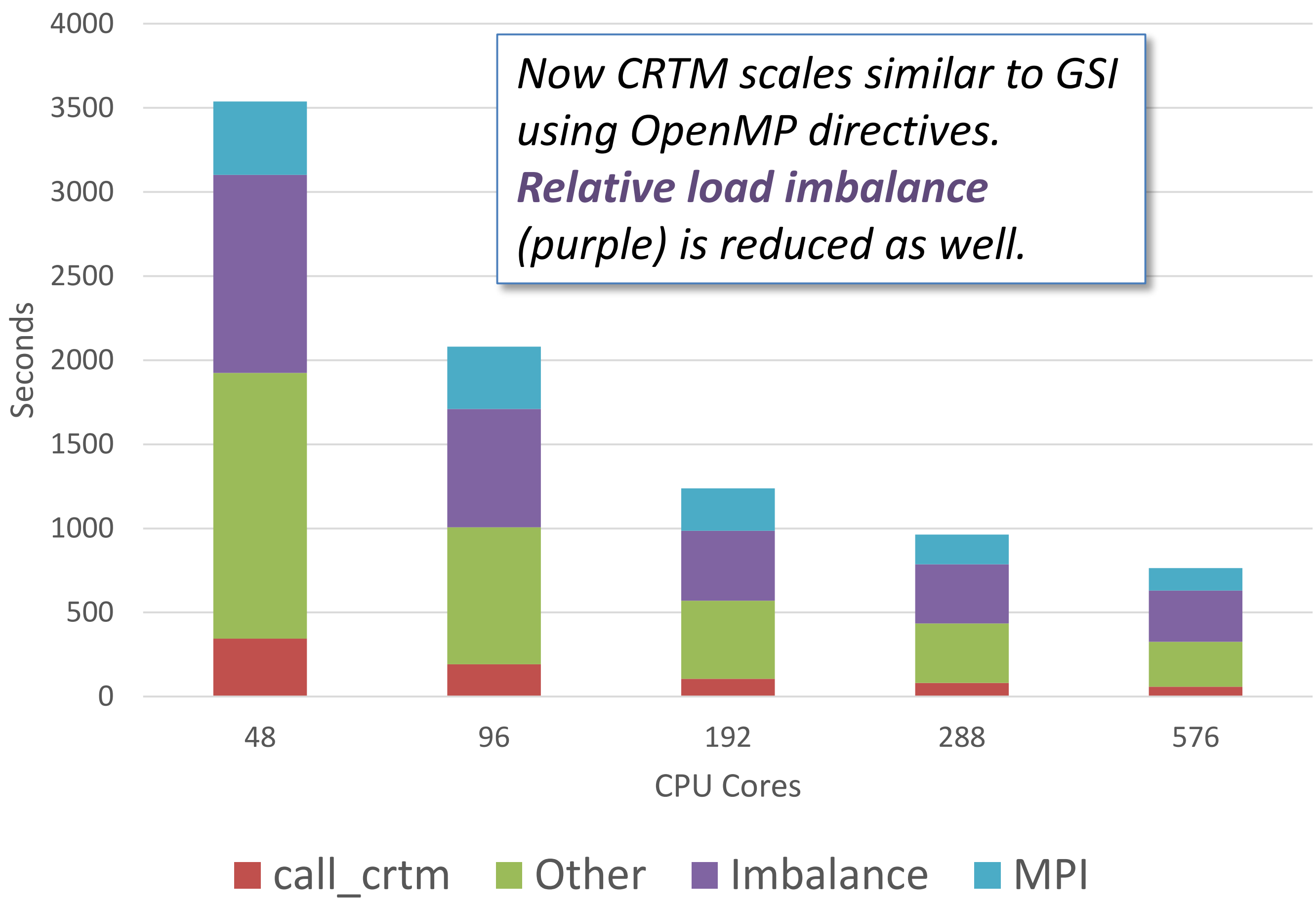
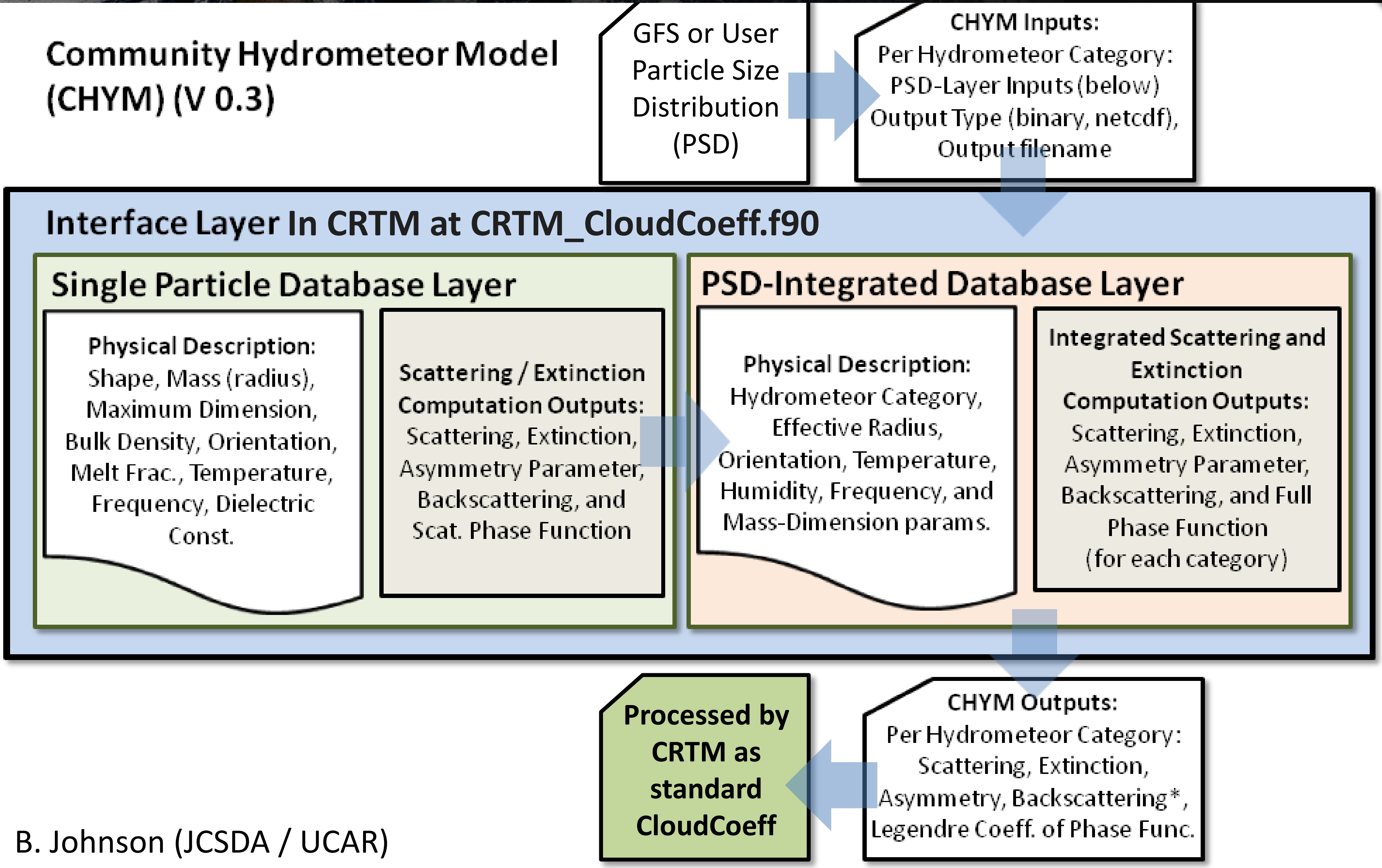


Community Hydrometeor Model (CHYM)

Community Hydrometeor Model (CHYM) (V 0.3)



B. Johnson (JCSDA / UCAR)

Community Hydrometeor Model CHYM

- (1) **Development of the microphysical parameters of clouds and precipitation** (Lead: Emily Liu)
  - Relate to the current and planned GFS microphysical assumptions.
  - converting mixing ratios into particle size distributions (PSD) and habit distributions, consistent with the microphysics schemes
- (2) **Creating the PSD-integrated scattering properties** (Lead: Ben Johnson).
  - Extend and replace current CloudCoeff.bin lookup table, consistency with above microphysics
- (3) New: Addition of Aerosols to CHYM (similar to Clouds/ Precip. in structure)

Ice Crystal Model

User Input

- Size Distribution $n(D)$
- Characteristic Diameter D_e
- Mass-Dimension Relationship $m(D) = aD^b$
- Ice Water Content $w_x = \rho_a q_x$
- Number Concentration N_t

Single Ice Crystal
Microphysical and Optical Properties
Data Base

- Size Distribution
- Habit Distribution
- Convolve each single particle optical property with the size and habit distribution to obtain distribution mean (bulk) ice particle optical property
- Parameterize distribution mean (bulk) ice particle optical properties as a function of characteristic diameter:

$k_{ext}(D_e,\nu), k_{sca}(D_e,\nu), \omega_o,\ g(D_e,\nu), P(\Theta, D_e,\nu)$

- Distribution Mean (Bulk) Ice Particle Optical Properties
- Ice Water Content

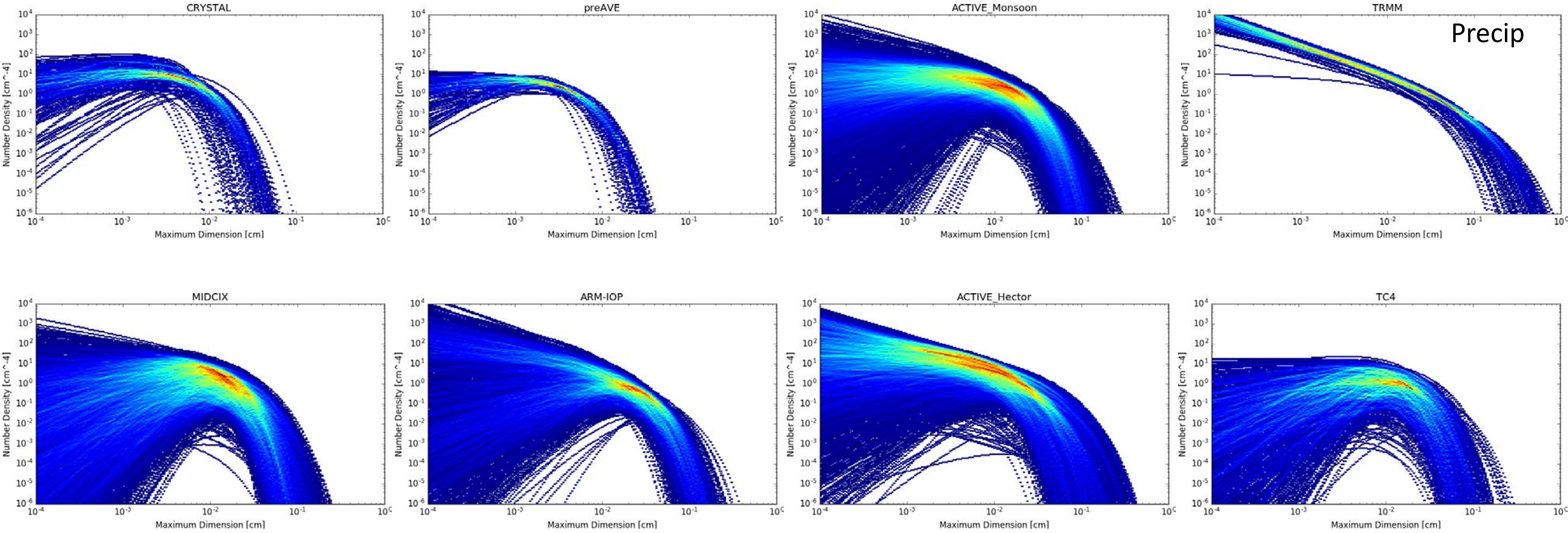
- Is the output IWC approximately equal to the input ?
- There is no doubt that mixture of habit is more realistic, but are we using the habit distribution which best represents the nature ?
- Should we parameterize the cloud optical properties using the same ice crystal database and model for the radiation model used in GFS for consistency ?

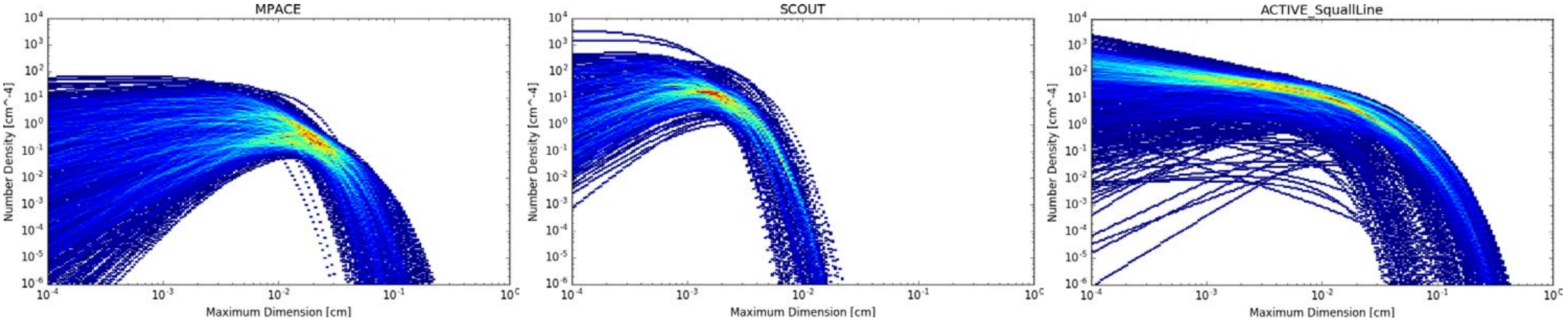
Field Campaign information

Field Campaign	Year	Location	Instruments	# PSDs
ARM-IOP	2000	Oklahoma, USA 2000	2D-C, 2D-P, CPI, CVI, FSSP	1420
TRMM-KWAJEX	1999	Kwajalein, Marshall Islands 1999	2D-C, HVPS, FSSP	201
CRYSTAL-FACE	2004	SE Florida/Caribbean 2002	CAPS (CIP, CAS), VIPS	62
SCOUT	2005	Darwin, Australia 2005	FSSP, CIP	553
ACTIVE – Monsoons	2005	Darwin, Australia 2005	CAPS (CIP, CAS)	4268
ACTIVE- Squall Lines	2005	Darwin, Australia 2005	CAPS (CIP, CAS)	740
ACTIVE-	2005	Darwin, Australia 2005	CAPS (CIP, CAS)	2583
MidCiX	2004	Oklahoma, USA 2004	CAPS (CIP, CAS), VIPS, FSSP	2968
Pre-AVE	2004	Houston,Texas, USA 2004	VIPS, CAPS	99
MPACE	2004	Alaska	2D-C	671
TC-4	2006	Costa Rica	CAPS, RIP	877

Credit: Brian Baum's website: http://www.ssec.wisc.edu/ice_models/microphysical_data.html

Observed Ice Particle Size Distributions

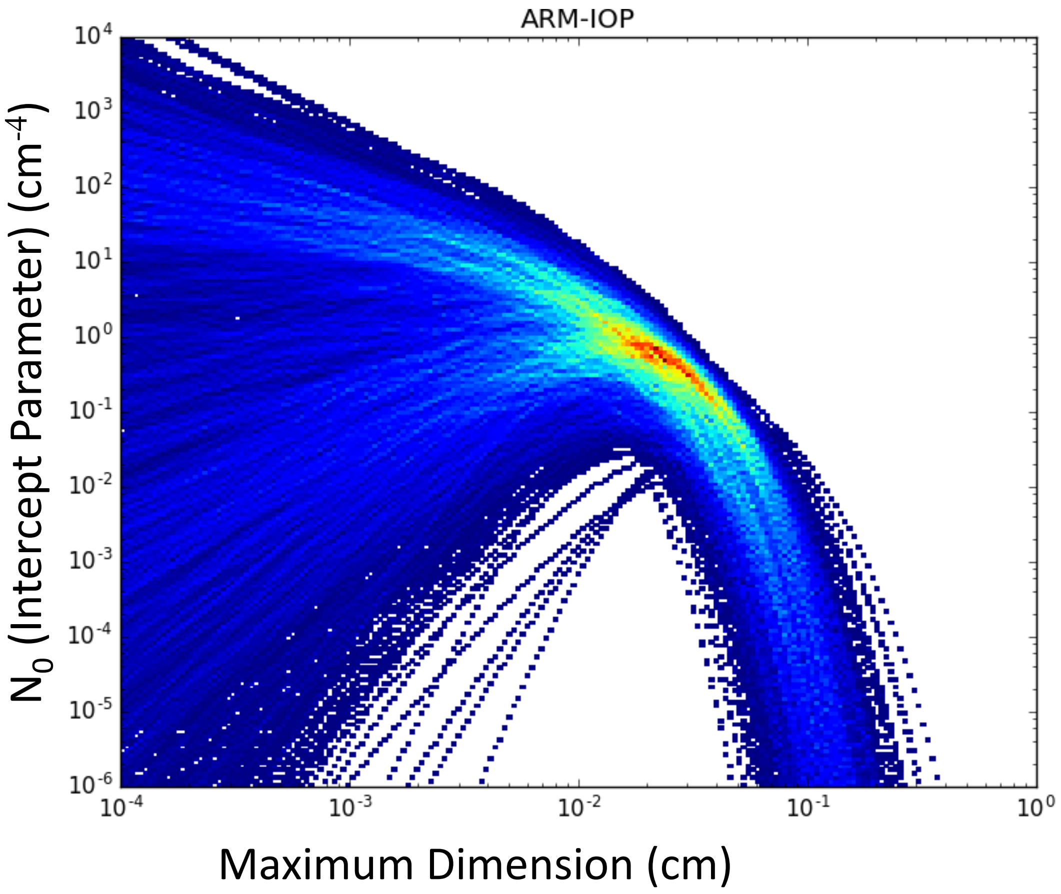


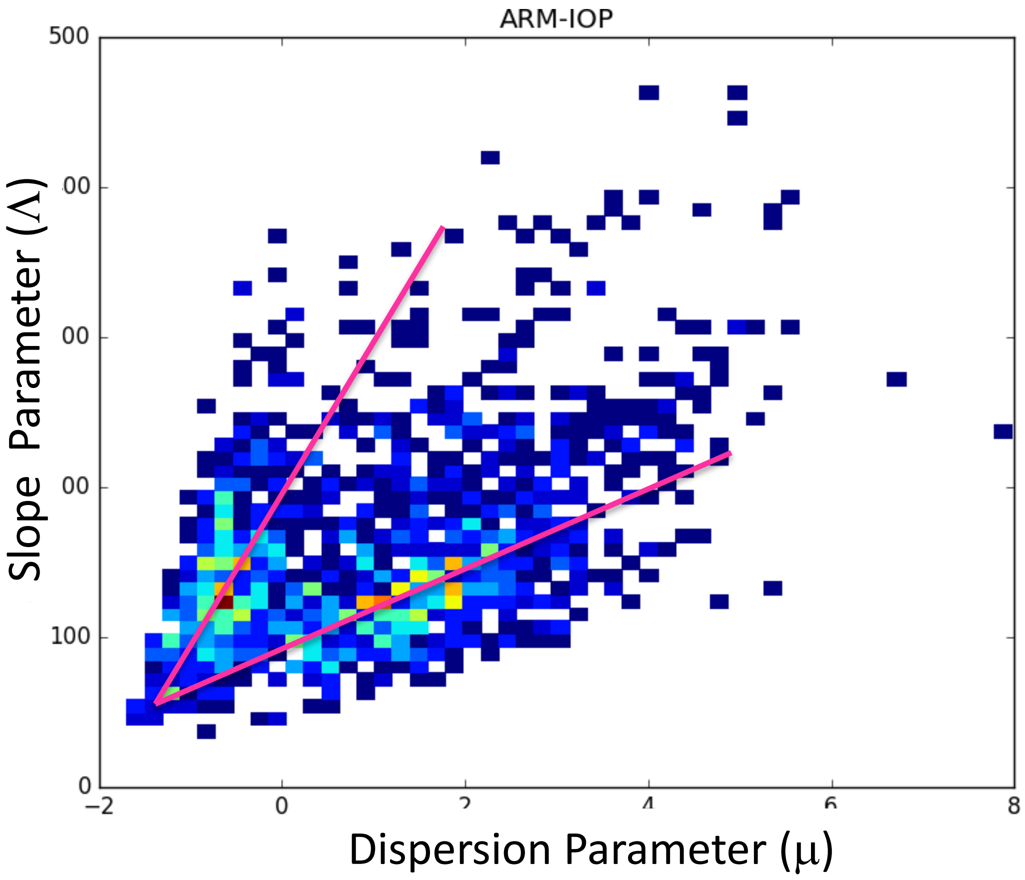


PSDs plotted using data downloaded from Brian Baum's website:
http://www.ssec.wisc.edu/ice_models/microphysical_data.htm

Cloud Physical Modeling (in CHYM)

Example: ARM Intensive Observation Program





3-parameter Gamma Distribution Function

General Gamma Function $n(D) = N_o\, D^{\mu}\, e^{-\lambda D^{\gamma}}$

3-parameter Gamma Function:

$n(D) = \mathbf{N_o}\, D^{\boldsymbol{\mu}}\, e^{-\boldsymbol{\lambda} D}$ where $\gamma = 1$; λ is the slope, μ is the dispersion, and N_o is the intercept when $\mu = 0$
D is maximum dimension

Some Useful Expressions related to Gamma Function

$$M_k = \int_0^{\infty} D^k\, n(D)\, dD = N_o \int_0^{\infty} D^{k+\mu}\, e^{-\lambda D}\, dD = N_o\, \Gamma(\mu + k + 1)\, \lambda^{-(\mu+k+1)} \; where\;\; \Gamma(x) = (x-1)!$$

k^th Moment of 3-parameter Gamma Function

$m(D) = aD^b$ Mass and Max. Diameter Relationship

$$N_t = M_0 = \int_0^{\infty} n(D)\, dD = N_o \int_0^{\infty} D^{\mu}\, e^{-\lambda D}\, dD = N_o\, \Gamma(\mu + 1)\, \lambda^{-(\mu+1)}$$

Total Particle Number Concentration

$$w_x = \rho_a q_x = \int_0^{\infty} m(D) n(D)\, dD = a \int_0^{\infty} D^b n(D) dD = a\, M_b = a\, N_o \int_0^{\infty} D^b\, e^{-\lambda D}\, dD = a\, N_o\, \Gamma(\mu + b + 1)\, \lambda^{-(\mu+b+1)}$$

(Hydrometeor Water Content w_x = Density of Dry Air ρ_a x Hydrometeor Mixing Ratio q_x)

3-parameter Gamma Distribution Function

For **single-moment** species (hydrometeor mixing ratio q_x is prognostic) :

N_{ox} is either fixed or prescribed as a function of temperature or mixing ratio

μ is set to zero for exponential distribution (Marshall-Palmer) or prescribed

λ, the slope can be calculated from hydrometeor mixing ratio q_x as:

$$w_x = \rho_a q_x = a\, N_{ox}\, \Gamma(\mu + b + 1)\, \lambda^{-(\mu+b+1)} \longrightarrow \lambda = \left(\frac{a\, N_{ox}\, \Gamma(\mu + b + 1)}{\rho_a q_x} \right)^{\frac{1}{\mu+b+1}}$$

Mapping of single-moment model mixing ratio to PSD parameters

For **double-moment** species (both mixing ratio q_x and total number concentration N_{tx} are prognostic) :

μ is set to zero for exponential distribution (Marshell-Palmer) or prescribed

N_{0x}, the intercept can be calculated from N_{tx} as:

$$N_{tx} = N_{ox}\, \Gamma(\mu + 1)\, \lambda^{-(\mu+1)} \longrightarrow N_{ox} = \frac{N_{tx}\, \lambda^{\mu+1}}{\Gamma(\mu + 1)}$$

λ, the slope can be calculated from N_{tx} and q_x as:

$$w_x = \rho_a q_x = a\, N_{ox}\, \Gamma(\mu + b + 1)\, \lambda^{-(\mu+b+1)} \longrightarrow \lambda = \left(\frac{a\, N_{tx}\, \Gamma(\mu + b + 1)}{\Gamma(\mu + 1)\, \rho_a q_x} \right)^{\frac{1}{b}}$$

Mapping of double-moment concentration and mixing ratio to PSD parameters

Field 07 Snow Particle Size Distribution

IWC

T_c

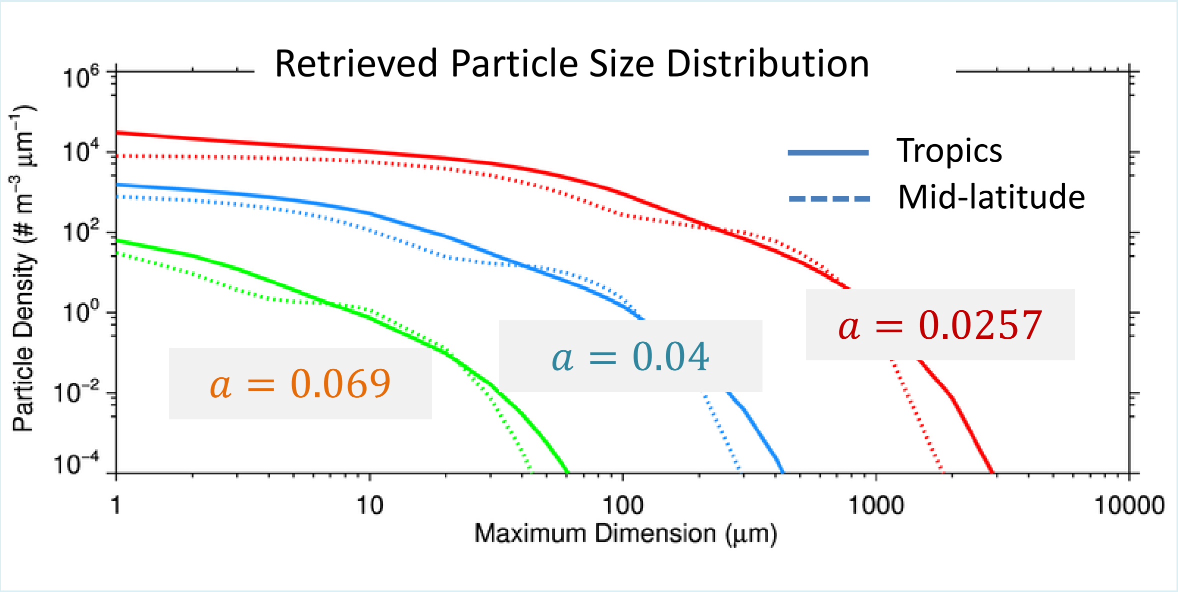
Mass-Dimensional Relationship

$$m(D) = aD^b$$

$$IWC = 0.0931\ gm^{-3}$$

$$T_c = -49.7\ °C$$

$$m(D) = aD^2$$



F07 PSD Parameterization

- Based on 10,000 in situ PSDs
- Second moment is linked to any other moment via polynomial fits to the in-cloud temperature
- Any given IWC and in-cloud temperature, the original PSD can be estimated

$$w_s = \rho_a\, q_s = a_s\, M_{b_s} \rightarrow M_{b_s} = \frac{w_s}{a_s} = \frac{\rho_a\, q_s}{a_s}$$

$$M_2 = \left(\frac{M_{b_s}}{A(b_s)\ exp[B(b_s)T_c]}\right)^{\frac{1}{C(b_s)}} \rightarrow M_n = A(n)\ \exp[B(n)T_c]\ M_2^{C(n)}$$

$$\begin{cases} x = D\dfrac{M_2}{M_3} \\ \Phi_{23}(x) = N(D)\dfrac{M_3^3}{M_2^4} \end{cases}$$

$$N(D) = \Phi_{23}(x)\frac{M_2^4}{M_3^3}$$

$$M_n = A(n)\ \exp[B(n)T_c]\ M_2^{C(n)}$$

$$A(n) = exp(13.6 - 7.76n + 0.479n^2)$$

$$B(n) = -0.0361 + 0.0151n + 0.00149n^2$$

$$C(n) = 0.807 + 0.00581n + 0.0457n^2$$

Tropical Regime:

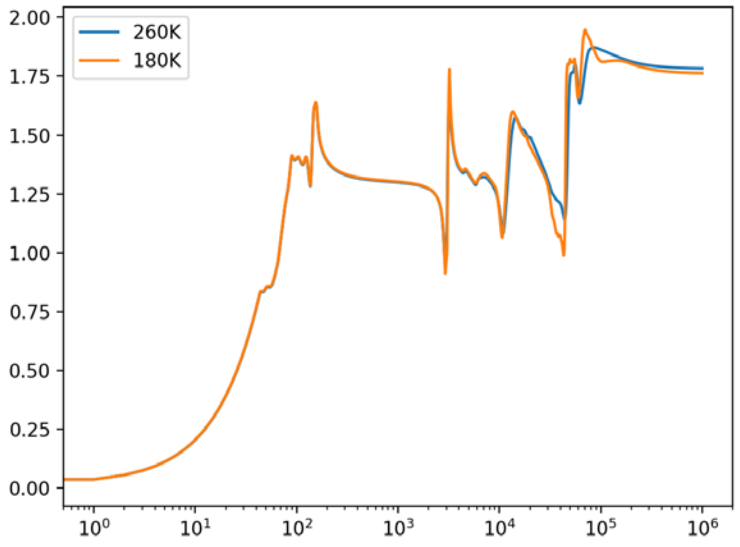
$$\Phi_{23}(x) = 152e^{-12.4x} + 3.28x^{-0.78}e^{-1.94x}$$

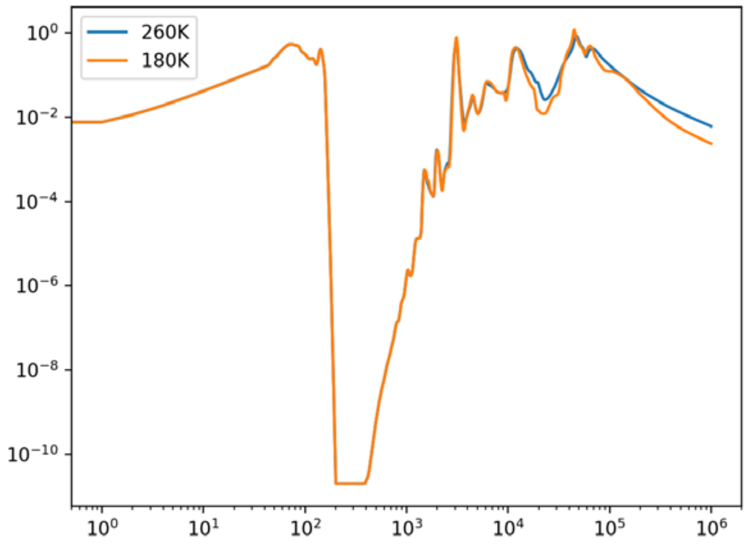
Mid-latitude Regime:

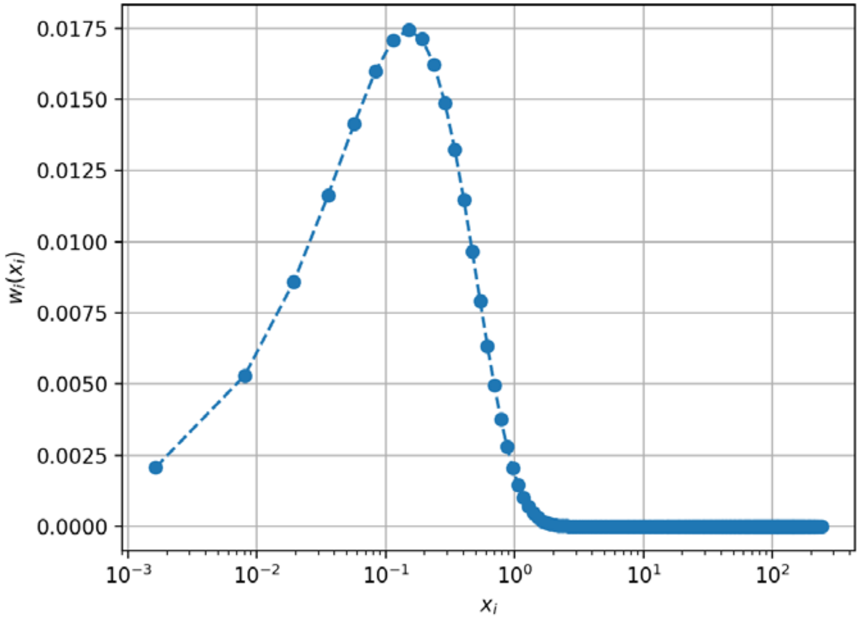
$$\Phi_{23}(x) = 141e^{-16.8x} + 102x^{2.07}e^{-4.82x}$$

CRTM Cloud / Precip. Scattering Table Work

- *Rayleigh- and Henyey-Greenstein phase matrices* provide a first quick placeholder solution.
- Investigation of feasibility of using *Normalized Particle Size Distributions* for Bulk Scattering Properties (convenient alternative: MC6 PSD).
- Refractive index database of Iwabuchi and Yang (2011) included in single-scattering calculations.
- Advanced quadrature schemes show potential for decreasing computation time of single-scattering properties (Gauss-Laguerre, Sinh-Tanh, etc.)







CRTM Cloud / Precip. Scattering Table Work

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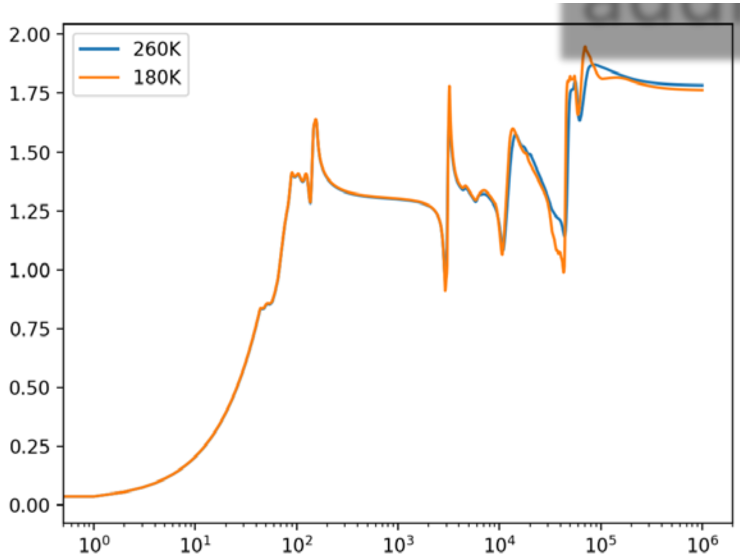
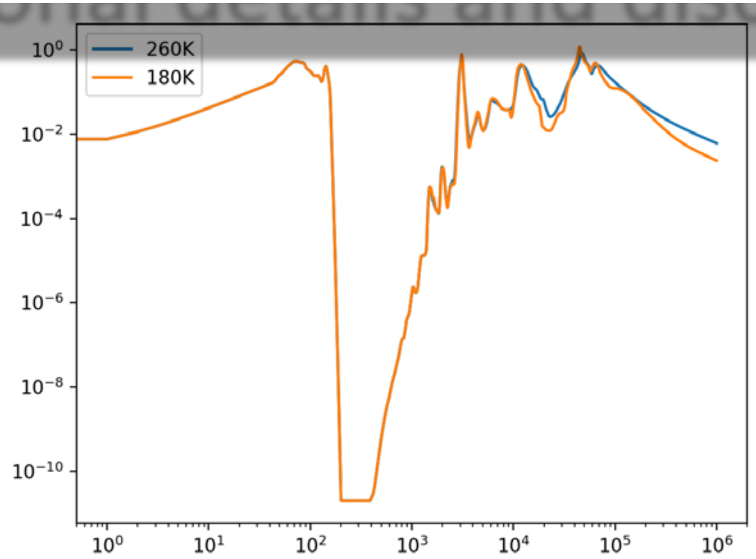
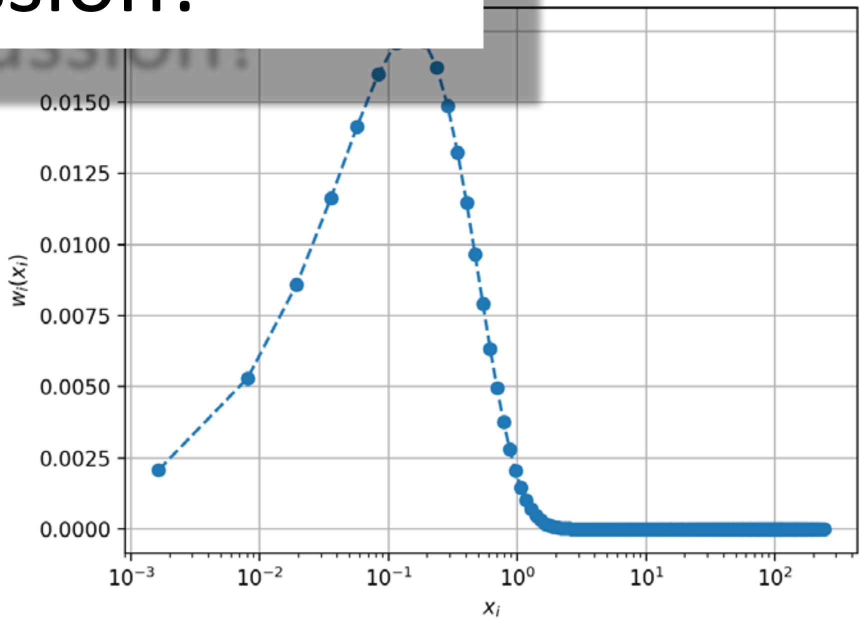
- Investigation of feasibility of using *Normalized Particle Size Distributions* for Bulk Scattering Properties (convenient alternative: MC6 PSD).
- Refractive index database of Iwabuchi and Yang (2011) included in single-scattering calculations.
- Advanced quadra … ingle-scattering proper…

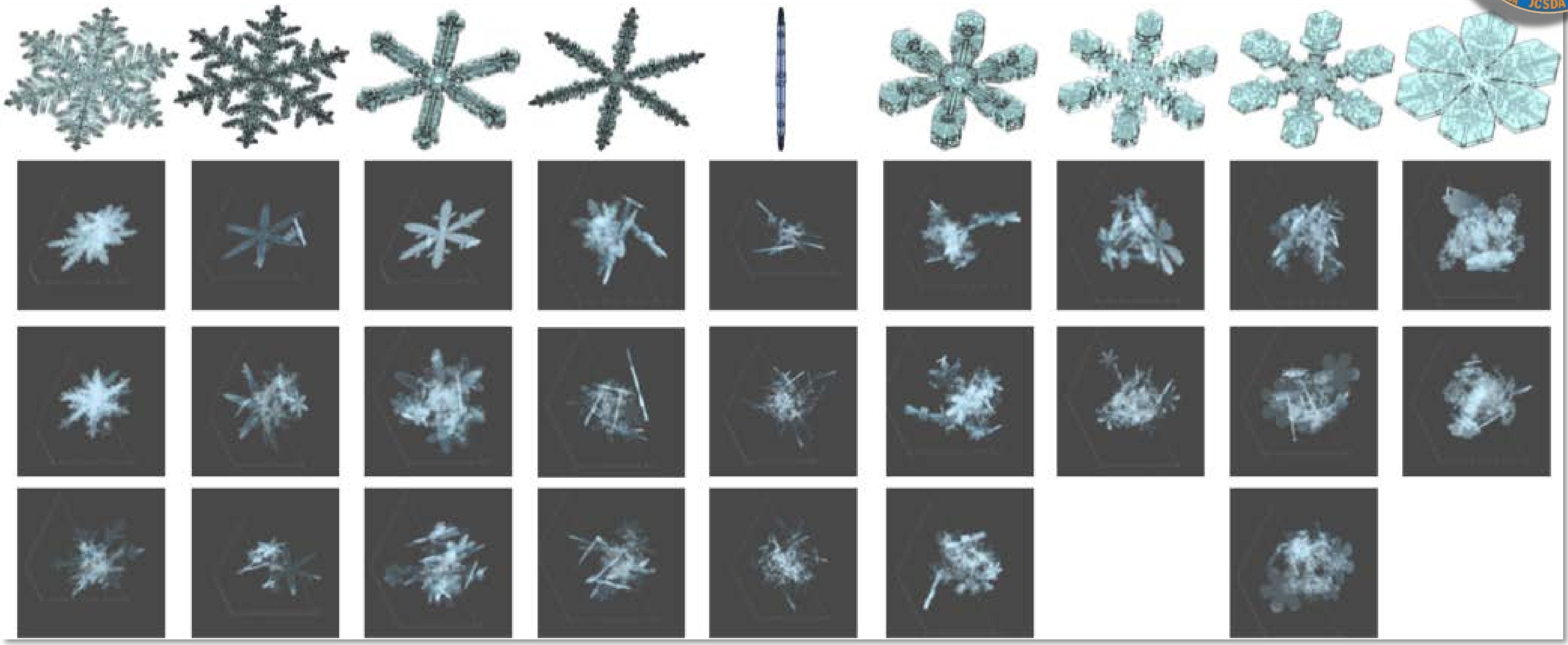
See Patrick Stegmann’s Poster for additional details and discussion!







CRTM Cloud / Precip. Scattering Table Work



The physical database contains 2126 pristine particle files, based on the above 9 base shapes, ranging from columns to plates to dendrites. Effective radius ranges from 60 to 1000 microns. The aggregate particle database, based on aggregates of the 9 base shapes above, consists of about 8100 aggregate shapes, with varying masses and constituent ice crystals. Effective radius ranges from 100 microns up to 5000 microns.

Enhancement of CRTM MODIS Collection 6 (MC6)

MODIS Collection 6

- A single habit ice model
- an ensemble of aggregates composed of eight severely roughened columns for ice cloud particles

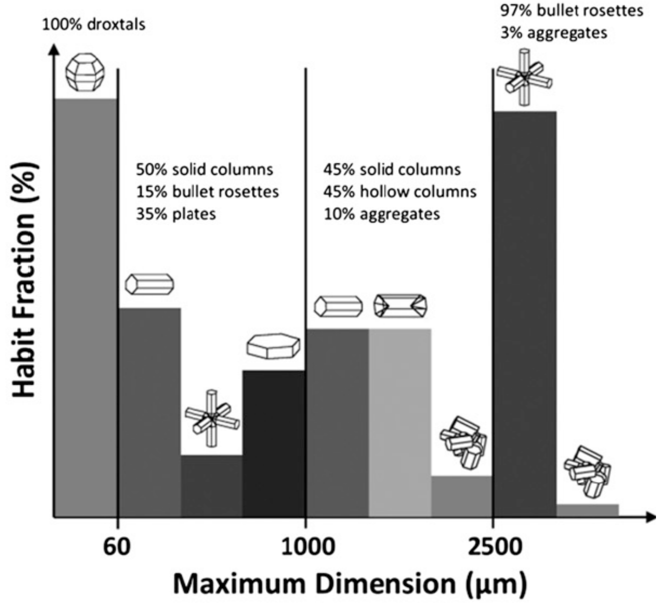
Single Particle Optical Properties

- Discrete Dipole Approximation (DDA) for small particles
- Geometric Optics (GO) Method for larger particles

Bulk Optical Properties

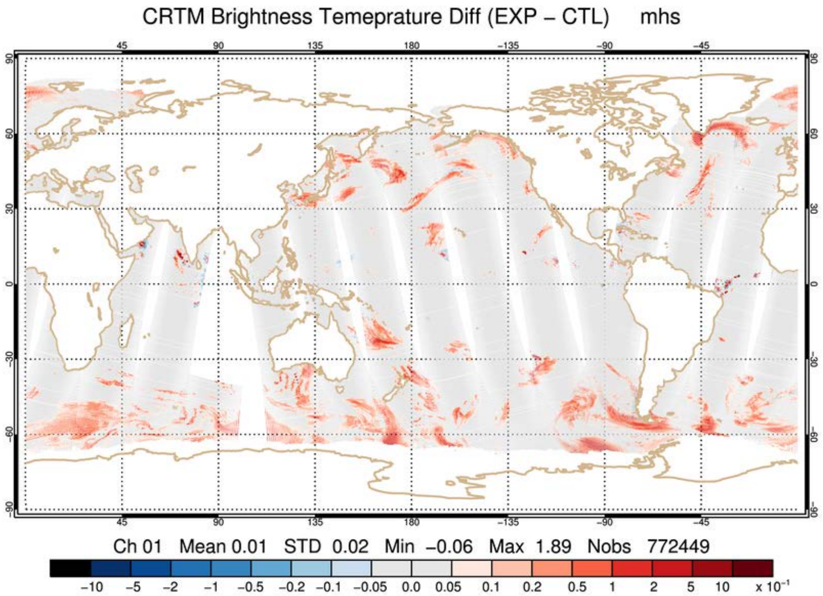
- Gamma size distribution
- Temperatures at 160K and 230K

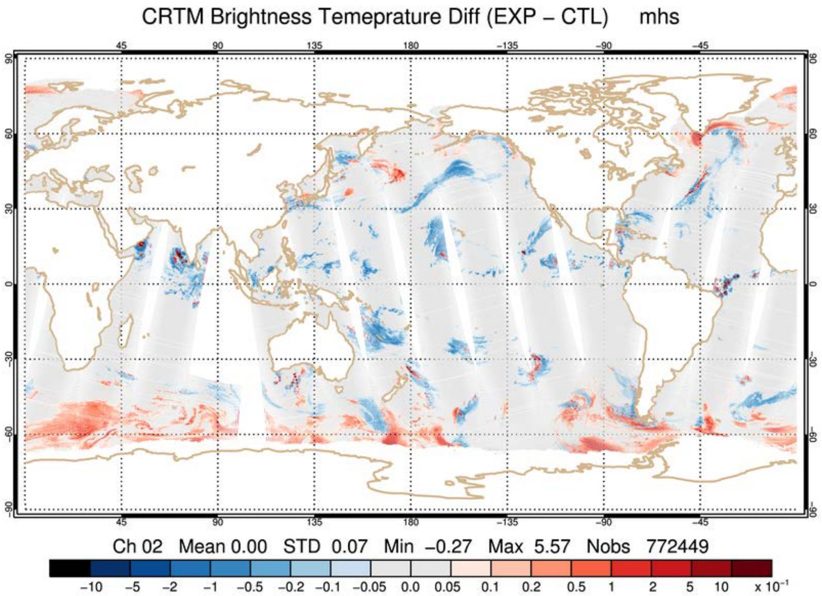
MODIS Collection 5



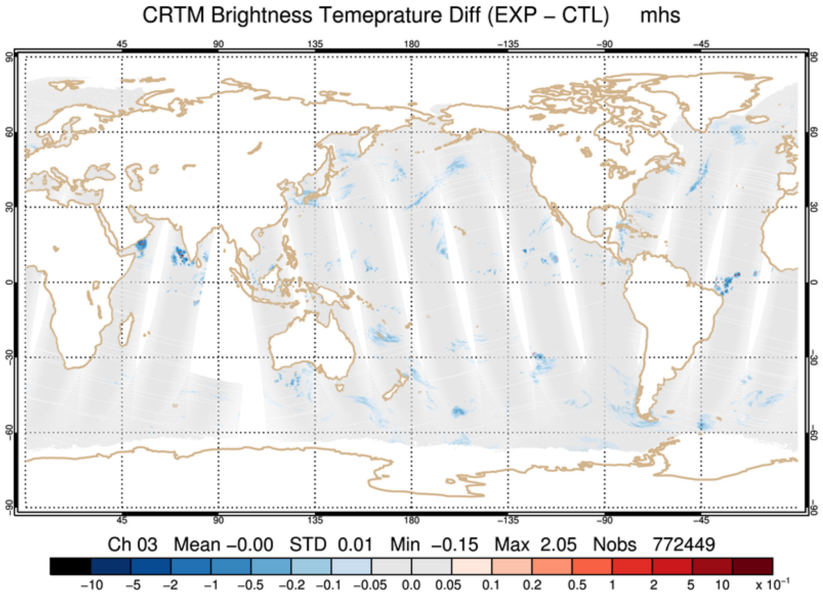
MODID Collection 6

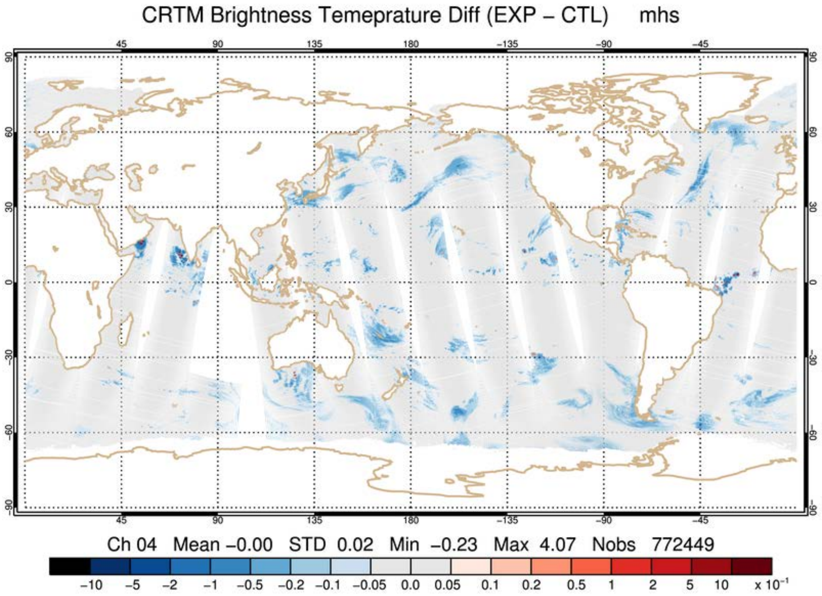
Enhancement of CRTM MC6 – Default

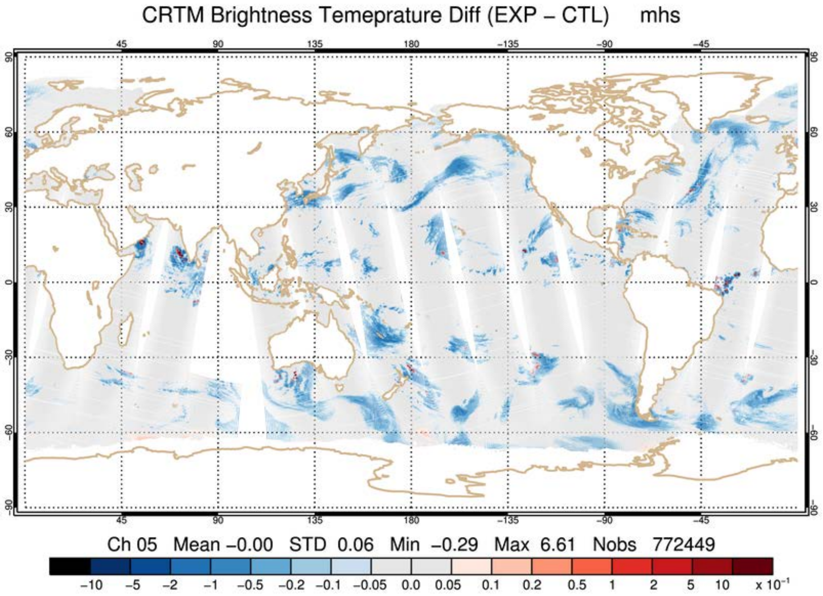




- MC6 Optical Properties evaluated at T=230 K
- Impact on AMSU-A is insignificant
- MHS Metop-a







CRTM 3: Surface Work :: CSEM (M. Chen)

Highlights:

1) CSEM top-down interfaces were refined to support upper-level vectorised RT solvers.

2) Integrated CRTM-CSEM version was successfully implemented in ProdGSI

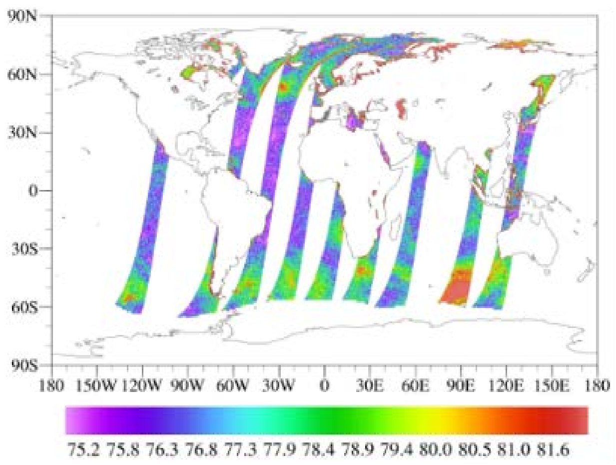
3) The tangent linear and adjoint modules of the physical MW land model implemented.

4) Implementation of L-band in CRTM has been tested with the integrated CRTM-CSEM.

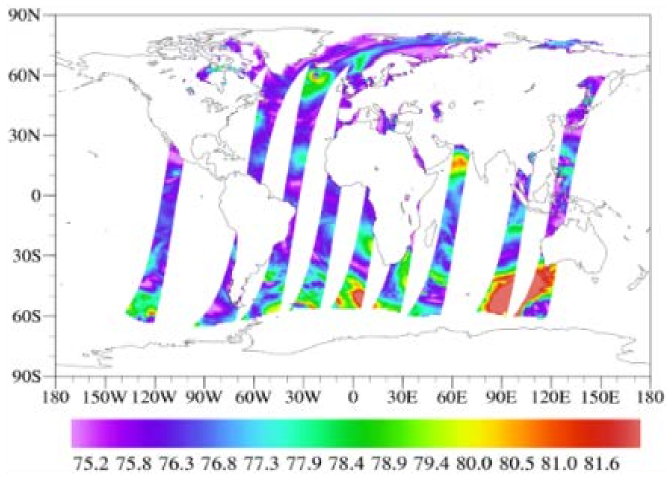
5) The testing of CRTM-CSEM in FV3 GFS/GSI is in progress.

6) Implementation of the JPL SMAP Level-3 monthly sea surface salinity (SSS) atlas into CSEM to account for the impact of SSS on the forward Tbs simulation and to improve the first guess accuracy in DA, especially for the L-band Tb.

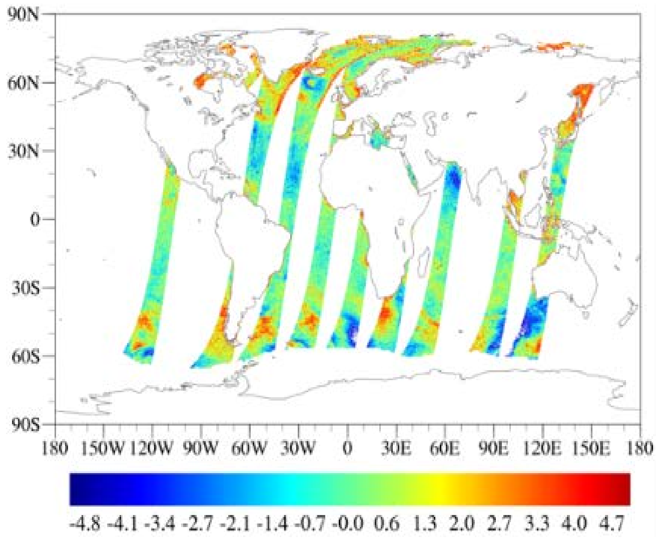
SMAP Observation

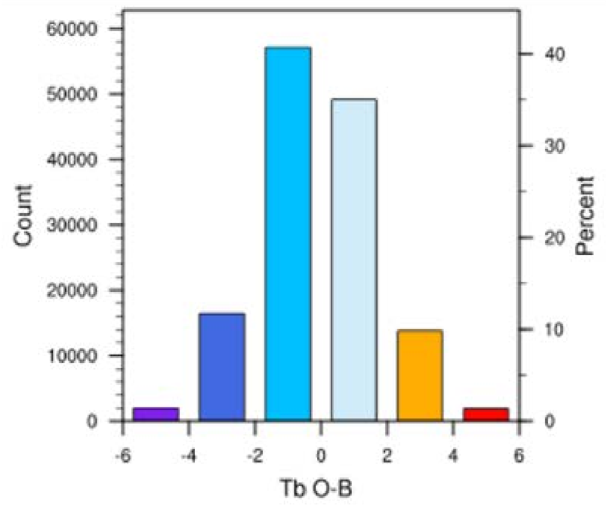


CRTM Simulation



SMAP-CRTM

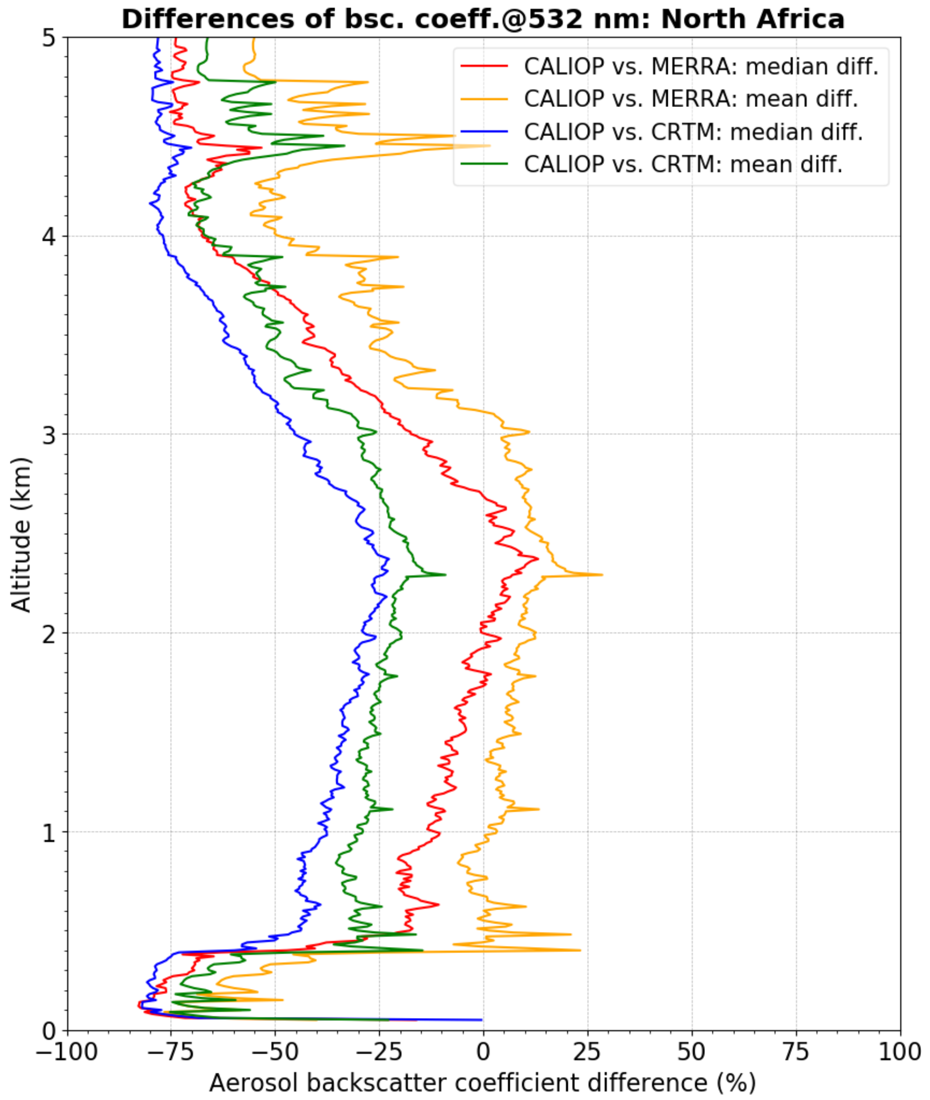




CRTM 3: Aerosol + Lidar Work

- Goal: Produce an aerosol-sensitive LIDAR forward operator for use in DA, initially focusing on CALIOP
- Output: Aerosol specific AOD and LIDAR backscattering coefficient.
- Status: Preliminary results (see fig.)
- CRTM backscattering compared to MERRA has similar variability, but is consistently too large.
- Future: update aerosol scattering tables, find source of difference.

Aerosol Backscattering Coefficient differences at 532 nm



Ongoing tasks toward CRTM 3.0

- Cloudy Radiance (Stegmann, E. Liu, Johnson)
  - Adding backscattering coefficients for CRTM active sensor capability.
  - Produce (Polarized) CRTM Scattering Coefficients from BHMIE and T-Matrix spheroids in binary and NetCDF
  - Start systematic investigation of “optimal” single-scattering properties for CRTM applications
- Surface (M. Chen, Y. Zhu)
  - Test CRTM-CSEM in GFS/GSI, focusing on the comparisons among model options.
  - Analyze and document the tests of CRTM-CSEM in GFS/GSI.
  - Initial implementation of MW ocean surface BRDF model.
  - Continued testing of CSEM in GSI
- Full Polarization Solver Capability (T. Greenwald, Q. Liu, B. Johnson, C. Cao)
  - UV capable solver + polarization support under development
  - Need to touch each element of CRTM to support UV capabilities – still establishing scope of effort required.
- SW / IR improvements in CRTM:
  - New UCAR hire expected within 6 months.
- Aerosols update (Johnson, Stegmann, S. Lu, M. Pagowski, B. Scherllin-Pirscher, Oyola/Ruston, others).
  - Update of CHYM to work with aerosol tables (Johnson, Stegmann)
  - Improved aerosol indices of refraction (via D. Turner and J. Gasteiger)
  - Update toward CMAQ specifications (Team)
  - Improve Lidar backscattering and attenuation calculations (Pagowski, Scherllin-Pirscher)

Education / Training / Outreach

The CRTM team successfully held the CRTM User/Developer’s workshop on May 16, 2017 in conjunction with the CRTM Scientific and Technical workshop (May 17 – May 19). The workshop consisted of a series of tutorials on CRTM operation and development. A particular focus was on covering the adjoint and tangent-linear programming. Also covered was spectral and transmittance coefficient generation, and regression / unit testing. There were 7 instructors more than 40 participants -- with about 16 in-person and more than 25 online. Feedback was overwhelmingly positive.

Questions / Comments?

- Possibility for a CGMS Satellite Data Assimilation Working Group?
- IDAWG
- Email: Benjamin.T.Johnson@noaa.gov

WRF with modification from Ruiyu Sun

THOMPSON CLOUD SCHEME

Liquid Hydrometeors

Variable	Habit	Density ρ	Mass-Diameter $m - D$	Size Distribution $N(D)$	Distribution Parameters	Effective (Characteristic) Diameter D_e
Cloud Water q_c $w_c = \rho_a q_c$	Spherical	1.00	$a_c D^{b_c} = \frac{\pi}{6}\rho_c D^3$	Gamma $N_{oc}\, D^{\mu_c}\, e^{-\lambda_c D}$	$N_{tc} = 10^8\ m^{-3}$ (maritime) Prescribed $\mu_c = min\left(15, \frac{10^9}{N_{tc}} + 2\right);\ 2 < \mu_c <= 15$ $\lambda_c = \left(\frac{a_c N_{tc}\, \Gamma(\mu_c + b_c + 1)}{\rho_a\, q_c\, \Gamma(\mu_c + 1)}\right)^{\frac{1}{b_c}}$ $N_{oc} = \frac{N_{tc}\, \lambda_c^{\mu_c+1}}{\Gamma(\mu_c+1)}$	$D_{ec} = \frac{M_3}{M_2}$ $= \frac{\int_0^\infty D^3 N(D) dD}{\int_0^\infty D^2 N(D) dD}$ $= \frac{\Gamma(\mu_c + 4)\lambda^{-(\mu_c+4)}}{\Gamma(\mu_c+3)\lambda^{-(\mu_c+3)}}$ $= \frac{\mu_c + 3}{\lambda_c}$
Rain q_r N_{tr} $w_r = \rho_a q_r$	Spherical	1.00	$a_r D^{b_r} = \frac{\pi}{6}\rho_r D^3$	Exponential $N_{or}\, D^{\mu_r}\, e^{-\lambda_r D}$	$\mu_r = 0$ $\lambda_r = \left(\frac{a_r N_{tr}\, \Gamma(\mu_r + b_r + 1)}{\rho_a\, q_r\, \Gamma(\mu_r + 1)}\right)^{\frac{1}{b_r}}$ $N_{or} = \frac{N_{tr}\, \lambda_r^{\mu+1}}{\Gamma(\mu_r + 1)}$	$D_{er} = \frac{M_3}{M_2}$ $= \frac{\int_0^\infty D^3 N(D) dD}{\int_0^\infty D^2 N(D) dD}$ $= \frac{\Gamma(\mu_r+4)\lambda^{-(\mu_r+4)}}{\Gamma(\mu_r+3)\lambda^{-(\mu_r+3)}}$ $= \frac{3}{\lambda_r}$

All units are defined in SI units unless noted

Solid Hydrometeors

Variable	Habit	Density ρ	Mass-Diameter $m - D$	Size Distribution $N(D)$	Distribution Parameters	Effective (Characteristic) Diameter D_e
Cloud Ice $\mathbf{q_i}$ $\mathbf{N_{ti}}$ $\mathbf{w_i = \rho_a q_i}$	Spherical	0.89	$a_i D^{b_i} = \frac{\pi}{6} \rho_i D^3$	Exponential $N_{oi}\, D^{\mu_i}\, e^{-\lambda_i D}$	$\mu_i = 0$ $\lambda_i = \left(\frac{a_i\, N_{ti}\, \Gamma(\mu_i + b_i + 1)}{\rho_a\, q_i\, \Gamma(\mu_i + 1)} \right)^{\frac{1}{b_i}}$ $N_{oi} = \frac{N_{ti}\, \lambda_i^{\mu+1}}{\Gamma(\mu_i + 1)}$	$D_{ei} = \frac{M_3}{M_2}$ $= \frac{\int_0^\infty D^3 N(D) dD}{\int_0^\infty D^2 N(D) dD}$ $= \frac{\Gamma(\mu_i + 4) \lambda^{-(\mu_i + 4)}}{\Gamma(\mu_i + 3) \lambda^{-(\mu_i + 3)}}$ $= \frac{3}{\lambda_i}$

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Solid Hydrometeors

Variable	Habit	Density ρ	Mass-Diameter $m-D$	Size Distribution $N(D)$	Distribution Parameters	Effective (Characteristic) Diameter D_e
Graupel q_g $w_g = \rho_a q_g$	Spherical	0.50	$a_g D^{b_g} = \frac{\pi}{6}\rho_g D^3$	Exponential $N_{og}\, D^{\mu_g}\, e^{-\lambda_g D}$	$N_{o,min} = 10^{-4} \quad N_{o.max} = 3\times10^{6}$ $x = \begin{cases} 4.01 + \log_{10}(D_{med,r}) & T < 270.56 \text{ and } D_{med,r} < 10^{-4} \\ 0.01 & Otherwise \end{cases}$ $Where \;\; D_{med,r} = \frac{3+\mu_{0.}+0.672}{\lambda_r} \;\; is\ the\ median\ mass\ diameter\ for\ rain$ $y = 4.31 + \log_{10}(\max(5\times10^{-5}, \rho_a q_r))$ $z = 3.1 + \frac{100}{\left[\frac{300xy}{\left(\frac{10}{x}+1+0.25y\right)} + 30 + 10y\right]}$ $N_{o,exp} = 10^{z}$ $N_{o,exp} = max\left(N_{o,min}, min(N_{o,exp}, N_{o,max})\right)$ $N_{o,min} = min(N_{o,exp}, N_{o,min})$ $N_{o,exp} = N_{o,min}$ $\lambda_{exp} = N_{o,exp}\left(\frac{a_g\Gamma(b_g+1)}{\rho_a q_g}\right)^{\frac{1}{b_g+1}}$ $\lambda_g = \lambda_{exp}\left(\frac{\Gamma(b_g+\mu_g+1)}{(b_g+\mu_g+1)(\mu_g+1)}\right)^{\frac{1}{b_g}}$ $N_{og} = \frac{N_{o,exp}}{(\Gamma(\mu_g+1)\lambda_{exp})}\lambda_g^{\mu_g+1}$	$D_{eg} = \frac{M_3}{M_2}$ $= \frac{\int_0^\infty D^3 N(D)dD}{\int_0^\infty D^2 N(D)dD}$ $= \frac{\Gamma(\mu_g+4)\lambda^{-(\mu_g+4)}}{\Gamma(\mu_g+3)\lambda^{-(\mu_g+3)}}$ $= \frac{3}{\lambda_g}$

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Solid Hydrometeors (Field 2007)

Variable	Habit	Density ρ	Mass-Diameter $m - D$	Size Distribution $N(D)$	Distribution Parameters	Effective Diameter D_e
Snow q_s $w_s = \rho_a q_s$	Non-spherical Fractal-like aggregated crystals (Cox,1988)	Variable (?)	$a_s D^{b_s} = 0.069 D^2$	Field (2007) $\begin{cases} x = D\dfrac{M_2}{M_3} \\ \Phi_{23}(x) = N(D)\dfrac{M_3^3}{M_2^4} \end{cases}$ $N(D) = \Phi_{23}(x)\dfrac{M_2^4}{M_3^3}$	$M_n = A(n)\exp[B(n)T_c]\, M_2^{C(n)}$ $A(n) = exp(13.6 - 7.76n + 0.479n^2)$ $\mathrm{B}(n) = -0.0361 + 0.0151n + 0.00149n^2$ $C(n) = 0.807 + 0.00581n + 0.0457n^2$ $w_s = \rho_a\, q_s = a_s\, M_{b_s} \rightarrow M_{b_s} = \dfrac{w_s}{a_s} = \dfrac{\rho_a\, q_s}{a_s}$ $M_2 = \left(\dfrac{M_{b_s}}{A(b_s)\, exp[B(b_s)T_c]}\right)^{\frac{1}{C(b_s)}} \rightarrow M_n = A(n)\exp[B(n)T_c]\, M_2^{C(n)}$ Tropical Regime: $\Phi_{23}(x) = 152e^{-12.4x} + 3.28x^{-0.78}e^{-1.94x}$ Mid-latitude Regime: $\Phi_{23}(x) = 141e^{-16.8x} + 102x^{2.07}e^{-4.82x}$	$D_{es} = \dfrac{M_3}{M_2}$ $= \dfrac{\int_0^\infty D^3 N(D)dD}{\int_0^\infty D^2 N(D)dD}$

The effective ice-particle density is parameterized through a mass-dimension relationship

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